

Enhanced electromagnetic transition dipole moments and radiative decays of massive Majorana neutrinos thanks to the seesaw-induced non-unitary effects

Zhi-zhong Xing^{*} and Ye-Ling Zhou[†]

Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China

Abstract

In an extension of the standard electroweak theory where the phenomenon of lepton flavor mixing is described by a 3×3 unitary matrix V , the electric and magnetic dipole moments of three active neutrinos are suppressed not only by their tiny masses but also by the Glashow-Iliopoulos-Maiani (GIM) mechanism. We show that it is possible to lift the GIM suppression if the canonical seesaw mechanism of neutrino mass generation, which allows V to be slightly non-unitary, is taken into account. In view of current experimental constraints on the non-unitarity of V , we find that the effective electromagnetic transition dipole moments of three light Majorana neutrinos and the rates of their radiative decays can be maximally enhanced by a factor of $\mathcal{O}(10^2)$ and a factor of $\mathcal{O}(10^4)$, respectively. This novel observation reveals an intrinsic and presumably significant correlation between the electromagnetic properties of massive neutrinos and the origin of their small masses.

Typeset using REVTeX

^{*}E-mail: xingzz@ihep.ac.cn

[†]E-mail: zhouyeling@ihep.ac.cn

[1] The standard model (SM) of particle physics contains three species of massless neutrinos which only take part in weak interactions. Since 1998, a number of convincing neutrino oscillation experiments have established the fact that neutrinos are actually massive and lepton flavors mix in a way similar to the quark flavor mixing [1]. This breakthrough has lots of profound and far-reaching impacts on cosmology and astrophysics, as neutrinos are so abundant in the Universe that their tiny masses and flavor conversions may significantly affect the cosmic expansion, structure formation and many other astrophysical events [2].

The most popular mechanism of generating finite but tiny neutrino masses beyond the SM is the canonical seesaw mechanism [3], in which the small neutrino masses are attributed to the existence of heavy degrees of freedom such as the right-handed Majorana neutrinos. In this elegant picture the 3×3 lepton flavor mixing matrix V has a striking difference from the 3×3 quark flavor mixing matrix in the SM: it is not exactly unitary due to small mixing between light and heavy neutrinos as a result of the Yukawa interactions. The deviation of V from a unitary matrix can be appreciable, in particular when the seesaw mechanism works at the TeV scale so as to avoid the seesaw-induced hierarchy problem [4] and satisfy the testability requirement at the Large Hadron Collider [5]. A careful analysis of current data on the invisible width of the Z^0 boson, universality tests of electroweak interactions, rare leptonic decays and neutrino oscillations indicates that the unitarity of V is good at the percent level and possible non-unitary effects are also allowed at the same level [6].

Now that three active neutrinos acquire their respective masses in the seesaw mechanism, they should have the electromagnetic dipole moments (EMDMs) through quantum loops. The fact that the Majorana neutrinos are their own antiparticles implies that they can only have the *transition* EDMMs between two different neutrino mass eigenstates in an electric or magnetic field [2]. The relevant radiative decays of the heavier active neutrinos, which may contribute to the cosmic infrared background in the Universe [7], are of particular interest in cosmology. Since all the previous calculations of the EDMMs and radiative decays of massive neutrinos were done by assuming V to be exactly unitary, we find it highly necessary and important to recalculate the same quantities by taking into account the seesaw-induced non-unitary effects.

The aim of this paper is just to examine the seesaw-induced non-unitary effects on the EDMMs and radiative decays of active Majorana neutrinos. We do a complete one-loop calculation and demonstrate that such effects are likely to be significantly larger than the standard (unitary) contributions, because the suppression induced by the Glashow-Iliopoulos-Maiani (GIM) mechanism [8] in the latter case can now be lifted. A careful numerical analysis shows that the effective EDMMs of three neutrinos and the rates of their radiative decays can be maximally enhanced by a factor of $\mathcal{O}(10^2)$ and a factor of $\mathcal{O}(10^4)$. Such an intrinsic and presumably important correlation between the electromagnetic properties of massive neutrinos and the origin of their small masses must be taken seriously, and it may even serve as a sensitive touch-stone for the highly-regarded seesaw mechanism in the future.

[2] The canonical seesaw mechanism is based on a simple extension of the SM in which three heavy right-handed neutrinos are added and the lepton number is violated by their Majorana mass term [3]:

$$-\mathcal{L}_\nu = \bar{\ell}_L Y_\nu \tilde{H} N_R + \frac{1}{2} \bar{N}_R^c M_R N_R + \text{h.c.} , \quad (1)$$

where $\tilde{H} \equiv i\sigma_2 H^*$ with H being the SM Higgs doublet, ℓ_L denotes the left-handed lepton doublet, N_R stands for the column vector of three right-handed neutrinos, and M_R is a symmetric Majorana mass matrix. After spontaneous $SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{\text{em}}$ gauge symmetry breaking, H achieves its vacuum expectation value $\langle H \rangle = v/\sqrt{2}$ with $v \simeq 246$ GeV. Then the Yukawa-interaction term in \mathcal{L}_ν yields the Dirac mass matrix $M_D = Y_\nu v/\sqrt{2}$, but the Majorana mass term in \mathcal{L}_ν keeps unchanged because right-handed neutrinos are the $SU(2)_L$ singlet and thus they are not subject to the electroweak symmetry breaking. The overall neutrino mass matrix turns out to be a symmetric 6×6 matrix and can be diagonalized through

$$\mathcal{U}^\dagger \begin{pmatrix} \mathbf{0} & M_D \\ M_D^T & M_R \end{pmatrix} \mathcal{U}^* = \begin{pmatrix} \widehat{M}_\nu & \mathbf{0} \\ \mathbf{0} & \widehat{M}_N \end{pmatrix}, \quad (2)$$

in which we have defined $\widehat{M}_\nu \equiv \text{Diag}\{m_1, m_2, m_3\}$ and $\widehat{M}_N \equiv \text{Diag}\{M_1, M_2, M_3\}$ with m_i and M_i being the physical masses of three light neutrinos ν_i and three heavy neutrinos N_i (for $i = 1, 2, 3$). The 6×6 unitary matrix \mathcal{U} is decomposed as [9]

$$\mathcal{U} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & U_0 \end{pmatrix} \begin{pmatrix} A & R \\ S & B \end{pmatrix} \begin{pmatrix} V_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}, \quad (3)$$

where $\mathbf{1}$ denotes the 3×3 identity matrix, U_0 and V_0 are the 3×3 unitary matrices, and A , B , R and S are the 3×3 matrices which characterize the correlation between the active or light neutrino sector (V_0) and the sterile or heavy neutrino sector (U_0). A full parametrization of \mathcal{U} in terms of 15 mixing angles and 15 CP-violating phases has been given in Ref. [9]. One may express the flavor eigenstates of three active neutrinos in terms of the mass eigenstates ν_i and N_i . In the mass basis of charged leptons and neutrinos, the leptonic weak charged-current (cc) and neutral-current (nc) interactions read

$$\begin{aligned} -\mathcal{L}_{\text{cc}} &= \frac{g}{\sqrt{2}} \left[\overline{\ell_{\alpha L}} \gamma^\mu (V_{\alpha i} \nu_{iL} + R_{\alpha i} N_{iL}) W_\mu^- + \text{h.c.} \right], \\ -\mathcal{L}_{\text{nc}} &= \frac{g}{2 \cos \theta_w} \left\{ \overline{\nu_{iL}} \gamma^\mu (V^\dagger V)_{ij} \nu_{jL} + \overline{N_{iL}} \gamma^\mu (R^\dagger R)_{ij} N_{jL} + \left[\overline{\nu_{iL}} \gamma^\mu (V^\dagger R)_{ij} N_{jL} + \text{h.c.} \right] \right\} Z_\mu, \end{aligned} \quad (4)$$

where α runs over e , μ or τ , $V = AV_0$ is responsible for the flavor mixing of active neutrinos ν_i , and R measures the strength of charged-current interactions of heavy neutrinos N_i (for $i = 1, 2, 3$) [10]. A small deviation of V from V_0 is actually characterized by nonvanishing R , as $VV^\dagger = AA^\dagger = \mathbf{1} - RR^\dagger$ holds. The exact seesaw relation between the masses of light and heavy neutrinos is $V \widehat{M}_\nu V^T + R \widehat{M}_N R^T = \mathbf{0}$, which signifies the correlation between neutrino masses and flavor mixing parameters.

Let us consider the radiative $\nu_i \rightarrow \nu_j + \gamma$ transition, whose electromagnetic vertex can be written as

$$\Gamma_{ij}^\mu(0) = \mu_{ij} (i \sigma^{\mu\nu} q_\nu) + \epsilon_{ij} (\sigma^{\mu\nu} q_\nu \gamma_5) \quad (5)$$

for a real photon satisfying the on-shell conditions $q^2 = 0$ and $q_\mu \varepsilon^\mu = 0$. In Eq. (5) ϵ_{ij} and μ_{ij} are the electric and magnetic *transition* dipole moments of Majorana neutrinos, and their sizes can be calculated via the proper vertex diagrams in FIG. 1 (weak cc interactions). The

γ - Z self-energy diagrams in FIG. 2 (weak nc interactions) do not have any *net* contribution to ϵ_{ij} and μ_{ij} , but we find that they play a very crucial role in eliminating the infinities because the divergent terms originating from FIG. 1 are unable to automatically cancel out in the presence of the seesaw-induced non-unitary effects (i.e., $R \neq \mathbf{0}$ and $V \neq V_0$) unless those divergent terms originating from FIG. 2 are also taken into account. This observation is new. It implies that the non-unitary case under discussion is somewhat different from the unitary case (i.e., $R = \mathbf{0}$ and $V^\dagger V = V_0^\dagger V_0 = \mathbf{1}$) discussed before in the literature [11], where the Feynman diagrams in FIG. 2 are forbidden and the divergent terms arising from FIG. 1 can automatically cancel out.

After a careful treatment of the infinities and non-unitary effects in our calculations [12], we arrive at

$$\begin{aligned}\mu_{ij} &= \frac{ieG_F}{4\sqrt{2}\pi^2} (m_i + m_j) \sum_{\alpha} F_{\alpha} \text{Im} (V_{\alpha i} V_{\alpha j}^*) , \\ \epsilon_{ij} &= \frac{eG_F}{4\sqrt{2}\pi^2} (m_i - m_j) \sum_{\alpha} F_{\alpha} \text{Re} (V_{\alpha i} V_{\alpha j}^*) ,\end{aligned}\quad (6)$$

where

$$F_{\alpha} = \frac{3}{4} \left[\frac{2 - \xi_{\alpha}}{1 - \xi_{\alpha}} - \frac{2\xi_{\alpha}}{(1 - \xi_{\alpha})^2} + \frac{2\xi_{\alpha}^2 \ln \xi_{\alpha}}{(1 - \xi_{\alpha})^3} \right] \quad (7)$$

with $\xi_{\alpha} \equiv m_{\alpha}^2/M_W^2$ (for $\alpha = e, \mu, \tau$) denotes the one-loop function. Although this result is *formally* the same as that obtained in Ref. [11], they are *intrinsically* different as the seesaw-induced non-unitary effects on μ_{ij} and ϵ_{ij} were not considered in the previous works. To see how important such non-unitary effects may be, let us make two analytical approximations. First, $F_{\alpha} \simeq 3(2 - \xi_{\alpha})/4$ holds to a good degree of accuracy for $\xi_{\alpha} \ll 1$. Second, $V = AV_0 \simeq V_0 - TV_0$ is also a good approximation for small non-unitary corrections to V_0 , where [9]

$$\begin{aligned}V_0 &= \begin{pmatrix} c_{12}c_{13} & \hat{s}_{12}^*c_{13} & \hat{s}_{13}^* \\ -\hat{s}_{12}c_{23} - c_{12}\hat{s}_{13}\hat{s}_{23}^* & c_{12}c_{23} - \hat{s}_{12}^*\hat{s}_{13}\hat{s}_{23}^* & c_{13}\hat{s}_{23}^* \\ \hat{s}_{12}\hat{s}_{23} - c_{12}\hat{s}_{13}c_{23} & -c_{12}\hat{s}_{23} - \hat{s}_{12}^*\hat{s}_{13}c_{23} & c_{13}c_{23} \end{pmatrix}, \\ T &= \begin{pmatrix} \frac{1}{2} \sum_{k=4}^6 s_{1k}^2 & 0 & 0 \\ \sum_{k=4}^6 \hat{s}_{1k}\hat{s}_{2k}^* & \frac{1}{2} \sum_{k=4}^6 s_{2k}^2 & 0 \\ \sum_{k=4}^6 \hat{s}_{1k}\hat{s}_{3k}^* & \sum_{k=4}^6 \hat{s}_{2k}\hat{s}_{3k}^* & \frac{1}{2} \sum_{k=4}^6 s_{3k}^2 \end{pmatrix}\end{aligned}\quad (8)$$

with $c_{ij} \equiv \cos \theta_{ij}$ and $\hat{s}_{ij} \equiv e^{i\delta_{ij}} \sin \theta_{ij}$ (here θ_{ij} and δ_{ij} are the mixing angles and CP-violating phases). Note that the light-heavy neutrino mixing angles θ_{ik} (for $i = 1, 2, 3$ and $k = 4, 5, 6$) are at most of $\mathcal{O}(0.1)$ [6], such that the deviation of V from V_0 is at the percent level or much smaller. Then we obtain

$$\sum_{\alpha} F_{\alpha} (V_{\alpha i} V_{\alpha j}^*) \simeq -\frac{3}{2} \sum_{\alpha} [(V_0)_{\alpha i} (TV_0)_{\alpha j}^* + (TV_0)_{\alpha i} (V_0)_{\alpha j}^*] - \frac{3}{4} \sum_{\alpha} [\xi_{\alpha} (V_0)_{\alpha i} (V_0)_{\alpha j}^*] . \quad (9)$$

The first and second terms on the right-hand side of this equation correspond to the non-unitary and unitary contributions, respectively. While the former is suppressed by $s_{ik}^2 \lesssim \mathcal{O}(10^{-2})$ (for $i = 1, 2, 3$ and $k = 4, 5, 6$) hidden in T , the latter is suppressed by $\xi_\alpha \lesssim 4.9 \times 10^{-4}$ (for $\alpha = e, \mu, \tau$) due to the GIM mechanism. We therefore draw a generic conclusion that the seesaw-induced non-unitary effects on ϵ_{ij} and μ_{ij} can be comparable with or even larger than the standard (unitary) contributions.

The rates of radiative $\nu_i \rightarrow \nu_j + \gamma$ decays are more sensitive to the non-unitarity of V , since they directly depend on $|\mu_{ij}|^2$ and $|\epsilon_{ij}|^2$. Namely,

$$\Gamma_{\nu_i \rightarrow \nu_j + \gamma} = \frac{(m_i^2 - m_j^2)^3}{8\pi m_i^3} (|\mu_{ij}|^2 + |\epsilon_{ij}|^2) \simeq 5.3 \times \left(1 - \frac{m_j^2}{m_i^2}\right)^3 \left(\frac{m_i}{1 \text{ eV}}\right)^3 \left(\frac{\mu_{\text{eff}}}{\mu_B}\right)^2 \text{ s}^{-1} \quad (10)$$

with $\mu_{\text{eff}} \equiv \sqrt{|\mu_{ij}|^2 + |\epsilon_{ij}|^2}$ for $\nu_i \rightarrow \nu_j + \gamma$ being the effective EMDMs and $\mu_B = e/(2m_e)$ being the Bohr magneton. The size of $\Gamma_{\nu_i \rightarrow \nu_j + \gamma}$ can be experimentally constrained by observing no emission of the photons from solar ν_e and reactor $\bar{\nu}_e$ fluxes. More stringent constraints on μ_{eff} come from the Supernova 1987A limit on neutrino decays and from the cosmological limit on distortions of the cosmic microwave background radiation (in particular, its infrared part): $\mu_{\text{eff}} < \text{a few} \times 10^{-11} \mu_B$ [13]. Now that more and more interest is being paid to the cosmic infrared background relevant to the radiative decays of massive neutrinos [7], it is desirable to evaluate μ_{eff} and $\Gamma_{\nu_i \rightarrow \nu_j + \gamma}$ on a well-defined theoretical ground, such as the canonical seesaw mechanism under discussion.

3 We proceed to numerically illustrate the non-unitary effects on μ_{eff} and $\Gamma_{\nu_i \rightarrow \nu_j + \gamma}$. In view of current neutrino oscillation data [14], we take $\Delta m_{21}^2 \simeq 7.6 \times 10^{-5} \text{ eV}^2$, $\Delta m_{32}^2 \simeq \pm 2.5 \times 10^{-3} \text{ eV}^2$, $\theta_{12} \simeq 34^\circ$, $\theta_{23} \simeq 45^\circ$ and $\theta_{13} \simeq 9^\circ$ [15] as our typical inputs. The mass scale of three neutrinos, the sign of Δm_{32}^2 and the values of three CP-violating phases of V_0 remain unknown. In our numerical calculation we consider both normal ($\Delta m_{32}^2 > 0$) and inverted ($\Delta m_{32}^2 < 0$) hierarchies by fixing the smallest neutrino mass to be 5 meV, and allow all the CP-violating phases to vary between zero and 2π . Those small active-sterile neutrino mixing angles in Eq. (8) are constrained by present experimental data [6] as follows:

$$\begin{aligned} T_{11} &< 5.5 \times 10^{-3}, & |T_{21}| &< 7.0 \times 10^{-5}, \\ T_{22} &< 5.0 \times 10^{-3}, & |T_{31}| &< 1.6 \times 10^{-2}, \\ T_{33} &< 5.0 \times 10^{-3}, & |T_{32}| &< 1.0 \times 10^{-2}. \end{aligned} \quad (11)$$

Note that all s_{ik} in T (for $i = 1, 2, 3$ and $k = 4, 5, 6$) are positive or vanishing. The CP-violating phases δ_{ik} are all allowed to vary from zero to 2π , but they must satisfy the above constraints together with the relations $VV^\dagger + RR^\dagger = \mathbf{1}$ and $V\widehat{M}_\nu V^T + R\widehat{M}_N R^T = \mathbf{0}$. It is in principle possible to determine the masses of three heavy Majorana neutrinos with the help of the exact seesaw relation, if all the other parameters are known [16]. Although such a prediction is in practice impossible, we find that the upper bound on the non-unitarity of V implies the existence of a lower bound on M_i . In other words, the values of M_i cannot be too small (i.e., they should be far away from those of m_i) so as to avoid too significant non-unitary effects. To assure that radiative corrections to the masses of three light neutrinos (via

the one-loop self-energy diagrams involving the heavy neutrinos) are sufficiently small (e.g., smaller than 0.5 meV) and stable, we simply assume that the masses of three heavy neutrinos are nearly degenerate [17] and not more than $\mathcal{O}(1)$ TeV. This assumption implies that we are concentrating on a limited and safe parameter space of the TeV seesaw mechanism, but it is instructive enough to reveal the salient features of the non-unitary effects on the effective EMDMs $\mu_{\text{eff}}(\nu_i \rightarrow \nu_j + \gamma)$ and the radiative decay rates $\Gamma_{\nu_i \rightarrow \nu_j + \gamma}$.

To present our numerical results in a convenient way, let us define

$$\varepsilon_{\text{uv}} \equiv \left[\sum_{k=4}^6 (s_{1k}^2 + s_{2k}^2 + s_{3k}^2) \right]^{1/2}, \quad (12)$$

which measures the overall strength of the unitarity violation of V , and $\varepsilon_{\text{uv}} \in [0, 0.15]$ is reasonably taken in our calculations. Namely, we allow each s_{ik} (for $i = 1, 2, 3$ and $k = 4, 5, 6$) to vary in the range $0 \leq s_{ik} < 0.15$. The numerical dependence of $\mu_{\text{eff}}(\nu_i \rightarrow \nu_j + \gamma)$ and $\Gamma_{\nu_i \rightarrow \nu_j + \gamma}$ on ε_{uv} is shown in FIGs. 3 and 4, respectively. Some discussions are in order.

(1) Switching off the non-unitary effects (i.e., $\varepsilon_{\text{uv}} = 0$), we obtain the effective electromagnetic dipole moments

$$\mu_{\text{eff}} \simeq \begin{cases} (0.8 \sim 3.0) \times 10^{-25} \mu_{\text{B}} & (\nu_2 \rightarrow \nu_1 + \gamma), \\ (0.8 \sim 1.5) \times 10^{-24} \mu_{\text{B}} & (\nu_3 \rightarrow \nu_1 + \gamma), \\ (1.1 \sim 2.1) \times 10^{-24} \mu_{\text{B}} & (\nu_3 \rightarrow \nu_2 + \gamma), \end{cases} \quad (13)$$

for the normal mass hierarchy with $m_1 \simeq 5$ meV; and

$$\mu_{\text{eff}} \simeq \begin{cases} (0.01 \sim 2.0) \times 10^{-24} \mu_{\text{B}} & (\nu_2 \rightarrow \nu_1 + \gamma), \\ (0.8 \sim 1.5) \times 10^{-24} \mu_{\text{B}} & (\nu_3 \rightarrow \nu_1 + \gamma), \\ (1.3 \sim 2.0) \times 10^{-24} \mu_{\text{B}} & (\nu_3 \rightarrow \nu_2 + \gamma), \end{cases} \quad (14)$$

for the inverted mass hierarchy with $m_3 \simeq 5$ meV, where the uncertainties mainly come from the unknown CP-violating phases δ_{12} , δ_{13} and δ_{23} . Such standard (unitary) results are far below the observational upper bound on μ_{eff} ($< \text{a few} \times 10^{-11} \mu_{\text{B}}$ [13]), but they serve as a good reference to the non-unitary effects on μ_{eff} being explored in this work.

(2) FIGs. 3 and 4 clearly show that μ_{eff} and $\Gamma_{\nu_i \rightarrow \nu_j + \gamma}$ can be maximally enhanced by a factor of $\mathcal{O}(10^2)$ and a factor of $\mathcal{O}(10^4)$, respectively, in particular when ε_{uv} approaches its upper limit as set by current experimental data. Note that the magnitude of $\mu_{\text{eff}}(\nu_2 \rightarrow \nu_1 + \gamma)$ may be strongly suppressed in the inverted neutrino mass hierarchy. The reason is rather simple: on the one hand, $m_1 \simeq m_2$ holds in this case, and thus $\epsilon_{12} \propto (m_2 - m_1)$ must be very small; on the other hand, μ_{12} depends on $\text{Im}(V_{\alpha 1} V_{\alpha 2}^*)$, so it can also be very small when the CP-violating phases are around zero or π . This two-fold suppression becomes severer for the decay rate $\Gamma_{\nu_2 \rightarrow \nu_1 + \gamma}$, because it is proportional to $(m_2 - m_1)^3 \mu_{\text{eff}}^2(\nu_2 \rightarrow \nu_1 + \gamma)$.

(3) Our numerical analysis shows that the results of μ_{eff} and $\Gamma_{\nu_i \rightarrow \nu_j + \gamma}$ are sensitive to the absolute neutrino mass scale for both normal and inverted mass hierarchies. For instance, $\mu_{\text{eff}}(\nu_2 \rightarrow \nu_1 + \gamma)$ and $\mu_{\text{eff}}(\nu_3 \rightarrow \nu_1 + \gamma)$ get enhanced when m_1 changes from zero to 5 meV in the normal mass hierarchy; while $\mu_{\text{eff}}(\nu_1 \rightarrow \nu_3 + \gamma)$ and $\mu_{\text{eff}}(\nu_2 \rightarrow \nu_3 + \gamma)$ are enhanced when m_3 changes from zero to 5 meV in the inverted mass hierarchy. This kind of sensitivity

is not so obvious if one only takes a look at the expressions of μ_{ij} and ϵ_{ij} in Eq. (6). The main reason is that a change of m_1 or m_3 requires some fine-tuning of the active-sterile neutrino mixing angles and CP-violating phases as dictated by the exact seesaw relation $V\widehat{M}_\nu V^T + R\widehat{M}_N R^T = \mathbf{0}$, leading to a possibly significant change of μ_{eff} . The dependence of $\Gamma_{\nu_i \rightarrow \nu_j + \gamma}$ on the absolute neutrino mass scale is somewhat more complicated, as one can see from Eq. (10).

(4) We find that the CP-violating phases play a very important role in fitting both the exact seesaw relation and Eq. (11). If the heavy neutrino masses M_i are not suppressed, then an appreciable value of ε_{uv} requires some fine cancellations in the matrix product $R\widehat{M}_N R^T$ such that sufficiently small m_i can be obtained from $V\widehat{M}_\nu V^T = -R\widehat{M}_N R^T$. On the other hand, we remark that it is actually unnecessary to require M_i to be around or above the electroweak scale. The seesaw-induced non-unitary effects on μ_{eff} and $\Gamma_{\nu_i \rightarrow \nu_j + \gamma}$ can be significant even if one allows one, two or three heavy neutrinos to be relatively light (e.g., at the keV mass scale). Such sterile neutrinos are interesting in particle physics and cosmology. Note that it is easier to satisfy the exact seesaw relation with an appreciable value of ε_{uv} by arranging M_i to lie in the keV, MeV or GeV range. This kind of low-scale seesaw scenarios [18] might be technically natural, but they have more or less lost the seesaw spirit. Of course, sufficiently large M_i and sufficiently small θ_{ik} can always coexist to make the seesaw mechanism work in a natural way, but in this traditional case the non-unitary effects are too small to have any measurable consequences at low energies.

It is also worth pointing out that the seesaw-induced non-unitary effects on μ_{ij} and ϵ_{ij} are rather different from the case of making a naive assumption of the flavor mixing between three active neutrinos and a few light sterile neutrinos [19]. The latter can directly break the unitarity of the 3×3 active neutrino mixing matrix V and then lift the GIM suppression associated with μ_{ij} and ϵ_{ij} . This kind of non-unitary effects are not constrained by the seesaw relation, and thus they are more arbitrary and less motivated from the point of view of model building.

[4] We have explored the seesaw-induced non-unitary effects on the electromagnetic transition dipole moments and radiative decays of three active neutrinos. We find that such effects are possible to be comparable with or larger than the standard (unitary) contributions, because the suppression induced by the GIM mechanism in the latter case can be lifted. Our numerical analysis has illustrated that the effective electromagnetic dipole moments of three neutrinos and the rates of their radiative decays can be maximally enhanced by a factor of $\mathcal{O}(10^2)$ and a factor of $\mathcal{O}(10^4)$, respectively, no matter whether the seesaw scale is around or below the TeV energy scale. This observation is new and nontrivial, and it clearly reveals an intrinsic and presumably important correlation between the electromagnetic properties of massive neutrinos and the origin of their small masses. Such a correlation may even serve as a sensitive touch-stone for the highly-regarded seesaw mechanism.

We thank W. Chao, Y.F. Li and S. Zhou for useful discussions. This work was supported in part by the National Natural Science Foundation of China under Grant No. 11135009.

REFERENCES

- [1] K. Nakamura *et al.* (Particle Data Group), J. Phys. G **37**, 075021 (2010).
- [2] See, e.g., Z.Z. Xing and S. Zhou, *Neutrinos in Particle Physics, Astronomy and Cosmology* (Zhejiang University Press and Springer-Verlag, 2011).
- [3] P. Minkowski, Phys. Lett. B **67**, 421 (1977); T. Yanagida, in *Proceedings of the Workshop on Unified Theory and the Baryon Number of the Universe*, edited by O. Sawada and A. Sugamoto (KEK, Tsukuba, 1979), p. 95; M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by P. van Nieuwenhuizen and D. Freedman (North Holland, Amsterdam, 1979), p. 315; S.L. Glashow, in *Quarks and Leptons*, edited by M. Lévy *et al.* (Plenum, New York, 1980), p. 707; R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).
- [4] F. Vissani, Phys. Rev. D **57**, 7027 (1998); J.A. Casas, J.R. Espinosa, and I. Hidalgo, JHEP **0411**, 057 (2004); A. Abada *et al.*, JHEP **0712**, 061 (2007).
- [5] For a brief review, see: Z.Z. Xing, Prog. Theor. Phys. Suppl. **180**, 112 (2009); and references therein.
- [6] S. Antusch *et al.*, JHEP **0610**, 084 (2006).
- [7] See, e.g., A. Mirizzi, D. Montanino, and P. Serpico, Phys. Rev. D **76**, 053007 (2007); S. Matsuura *et al.*, Astrophys. J. **737**, 2 (2011); S.H. Kim *et al.*, J. Phys. Soc. Jap. **81**, 024101 (2012).
- [8] S.L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D **2**, 1285 (1970).
- [9] Z.Z. Xing, Phys. Rev. D **85**, 013008 (2012).
- [10] Z.Z. Xing, Phys. Lett. B **660**, 515 (2008).
- [11] R.E. Shrock, Nucl. Phys. B **206**, 359 (1982); P.B. Pal and L. Wolfenstein, Phys. Rev. D **25**, 766 (1982); B. Kayser, Phys. Rev. D **26**, 1662 (1982); J.F. Nieves, Phys. Rev. D **26**, 3152 (1982).
- [12] Z.Z. Xing and Y.L. Zhou, a detailed description of our one-loop calculations will be presented in a more comprehensive paper elsewhere.
- [13] G.G. Raffelt, Phys. Rept. **320**, 319 (1999); C. Giunti and A. Studenikin, Phys. Atom. Nucl. **72**, 2089 (2009).
- [14] G.L. Fogli *et al.*, Phys. Rev. D **84**, 053007 (2011); T. Schwetz, M. Tortola, and J.W.F. Valle, New J. Phys. **13**, 109401 (2011).
- [15] F.P. An *et al.* (Daya Bay Collaboration), Phys. Rev. Lett. **108**, 171803 (2012); J.K. Ahn *et al.* (RENO Collaboration), arXiv:1204.0626.
- [16] Z.Z. Xing, Phys. Lett. B **679**, 255 (2009).
- [17] A. Pilaftsis, Z. Phys. C **55**, 275 (1992); S. Zhou, *PhD Thesis* (Institute of High Energy Physics, Beijing, 2009).
- [18] A. de Gouvea, J. Jenkins, and N. Vasudevan, Phys. Rev. D **75**, 013003 (2007).
- [19] The present experimental or observational hints for the possible existence of light sterile neutrinos at the sub-eV or eV mass scale remain preliminary and controversial. See, e.g., J. Kopp, M. Maltoni, and T. Schwetz, Phys. Rev. Lett. **107**, 091801 (2011).

FIGURES

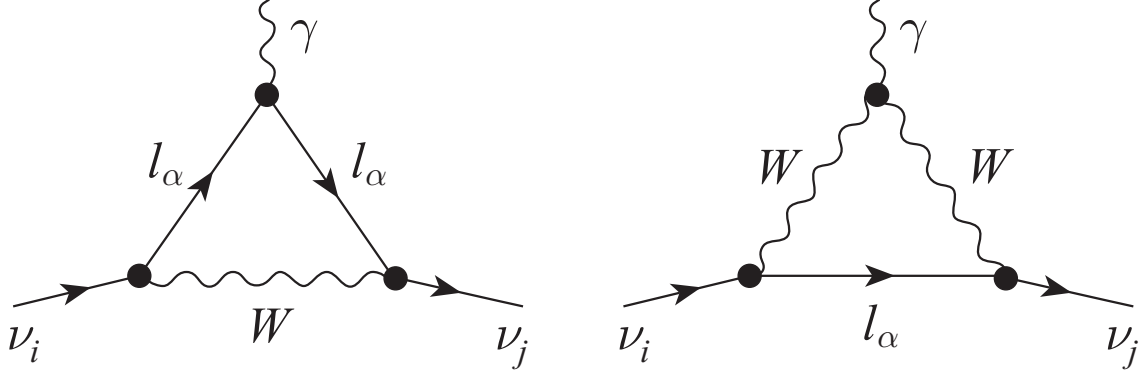


FIG. 1. The one-loop Feynman diagrams (and their charge-conjugate counterparts) contributing to the EMDMs of the Majorana neutrinos, where $\alpha = e, \mu, \tau$ and $i, j = 1, 2, 3$.

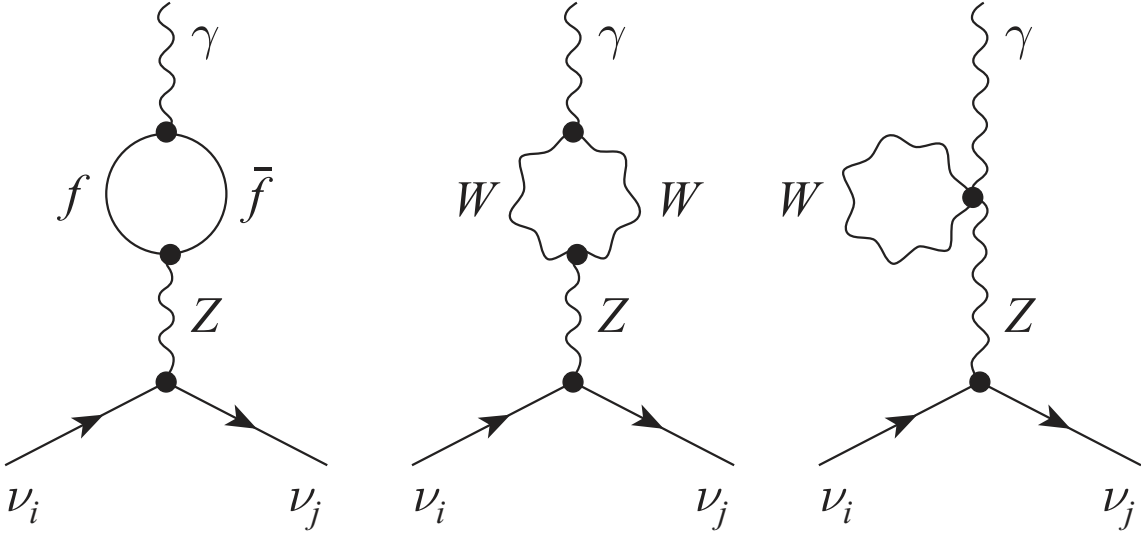


FIG. 2. The one-loop γ - Z self-energy diagrams (and their charge-conjugate counterparts) associated with the EMDMs of massive Majorana neutrinos, where f denotes all the SM fermions and $i, j = 1, 2, 3$.

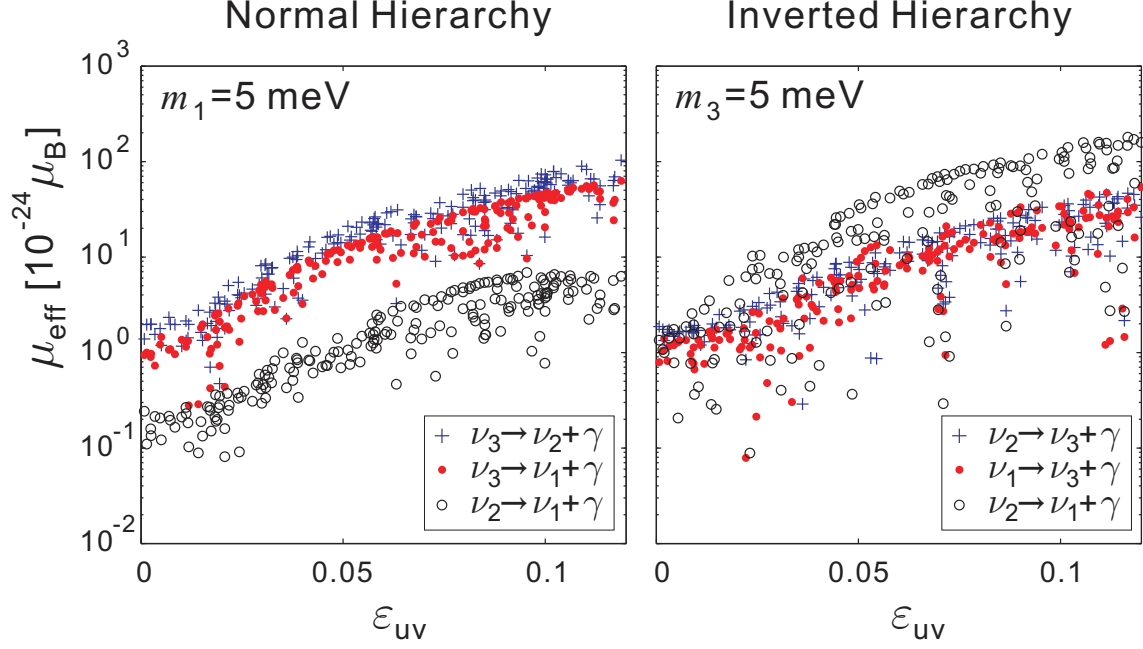


FIG. 3. Illustration of the seesaw-induced non-unitary effects on μ_{eff} for three active neutrinos. The standard (unitary) results correspond to $\varepsilon_{\text{uv}} = 0$, and their uncertainties come from the three unknown CP-violating phases of V_0 .

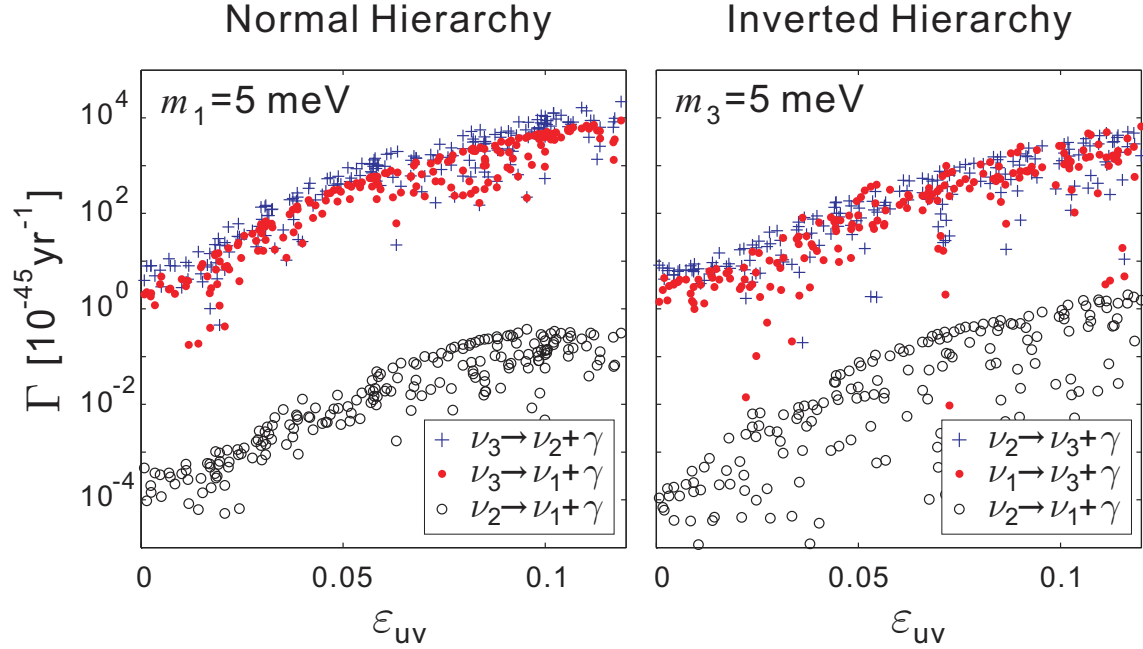


FIG. 4. Illustration of the seesaw-induced non-unitary effects on $\Gamma_{\nu_i \rightarrow \nu_j + \gamma}$ for three active neutrinos. The standard (unitary) results correspond to $\varepsilon_{\text{uv}} = 0$, and their uncertainties come from the three unknown CP-violating phases of V_0 .